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Shaft Power Transmission Systems for Fixed-Wing, V/STOL, GEM, and Hydrofoil Vehicles

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This paper presents some of the current technology for the design of flexible mechanical power transmission systems for fixed-wing, V/STOL, GEM, and hydrofoil craft. Many vehicles now require long power transmission systems, between the engine and the propellers or fans, that are lightweight and highly efficient. When choosing the optimum mechanical system, three different basic arrangements of flexible couplings, pillow blocks, and interconnecting tubing should be considered. Each of these systems has advantages and disadvantages regarding angular and axial movement capabilities; therefore, some estimate of the required over-all system flexibility is necessary prior to making the tradeoff studies discussed herein. In general, mechanical systems have extremely high over-all efficiency. The most efficient lightweight mechanical design results when the system is operated at the highest practical speed and optimum use is made of all materials. This is accomplished by minimizing sliding or rolling surfaces that require lubrication, choosing sections and shapes that are highly and uniformly stressed, and attempting to use the resulting elastic deflections to advantage. One approach to making system tradeoff studies and a discussion of various intermediate supports, splines, and the contoured diaphragm-type flexible couplings are included in this paper.

Nomenclature

E = modulus of elasticity, psi
l = shaft length, in.
N_c = first critical speed, rpm
S_b = bending stress, psi
S_s = torsional shear stress, psi
$S_{s'}$ = critical buckling shear stress, psi
r = outside radius of shaft, in.
r_i = inside radius of shaft, in.
T = torque, lb-in.
t = shaft wall thickness, in.
μ = Poisson's ratio
y = total deflection at end point, in.

Introduction

THE solution to the problem of transmitting power over long distances is of major interest because many of today's V/STOL's, GEM's, and hydrofoil craft have engines remotely mounted from the propellers or fans that they drive. When a power transmitting system is chosen, it must be lightweight, highly efficient, reliable, and have reasonable maintenance and overhaul periods. Weight is of extreme importance because these vehicles normally carry a very high weight penalty. High efficiency is a must. The very high over-all efficiency of a mechanical system as compared to electrical, pneumatic or hydraulic systems points to the mechanical-type system as the optimum-type system to employ.

Mechanical power transmission systems must be flexible because set-up tolerances, thermal movements, vibrations and dynamic movements between supports are always present.

When the engineer responsible for the design of a mechanical system considers what type to use, he must look into items such as: 1) angular misalignment (normal and transient), 2) axial movements (normal and transient), 3) horsepower (normal and transient), 4) length, and 5) speed. With some estimates of the direction and magnitude of these values, tradeoff studies of the various systems can be made to ascertain the optimum system for weight, size, efficiency, and reliability.

Three Basic Types of Mechanical Systems

A. Subcritical: Flexible Couplings at Ends Only

The system illustrated in Fig. 1 utilizes flex couplings at the end points and bending in the interconnecting tubing. Intermediate pillow blocks are required to keep the system operation in the subcritical range. These pillow blocks must be self aligning and in some instances must also be capable of axial movements. At least one telescoping member is required if the axial movements are in excess of the capability of the flexible couplings. A schematic of an actual type A system is shown in Fig. 2. The inserts in Fig. 2 are photographs of flexible couplings developed for this XC-142 V/STOL. From the schematic it can be seen that the air-

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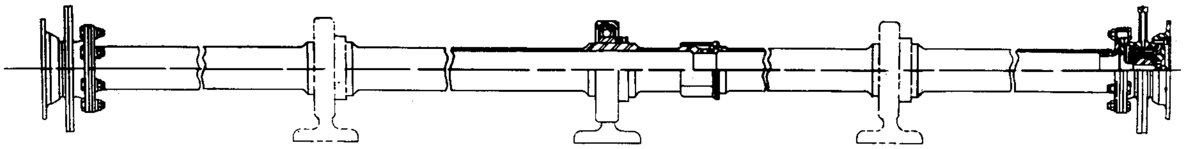

Fig. 1 Subcritical, type A, flexible couplings at ends only.

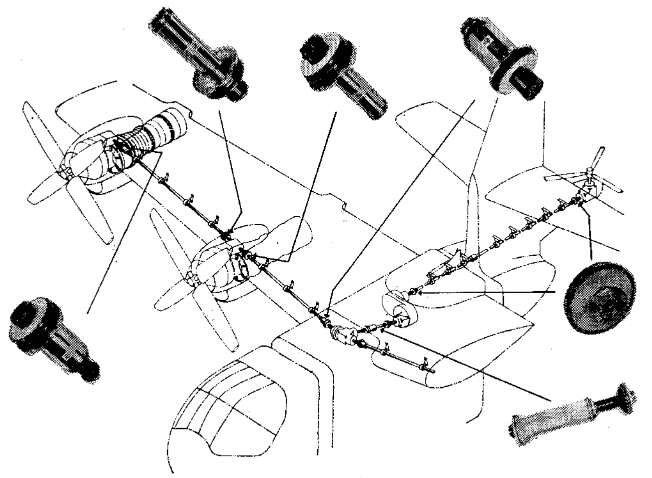

Fig. 2 System schematic XC-142 V/STOL.

craft is of the tilt-wing type. The engines are directly connected to the propellers and the integral gearboxes at each propeller are interconnected by the flexible shafting. The shaft system is capable of transmitting full power in case of an engine failure. Ball type splines are located in each major section of shafting to accommodate axial movements.

This type A subcritical system is becoming very popular because it requires very few complex components and can accommodate reasonable misalignments.

B. Subcritical: Flexible Couplings at Each Support

This type of system which has a flexible coupling at each pillow block as well as at the end point is illustrated in Fig. 3 and is used when an installation has high misalignments between pillow blocks and engines, gearboxes or propellers. The intermediate bearing pillow blocks must still have angular or self-aligning capabilities, and if axial movements cannot be accommodated by the flexible couplings, additional provisions must be made by including at least one axial telescoping spline member. In this case the bearing supports must also be capable of moving axially. One example of this type of system is shown in Fig. 4 and is an actual photograph of the Hiller OH-5A tail rotor system.

C. Hypercritical: Flexible Couplings at Ends Only

The preceding discussion has been limited to "subcritical" mechanical power transmission system or systems operating below their first natural (lateral) frequency (critical speed). The studies of these systems show that operation at moderately high speeds with relatively small diameter interconnecting shafting requires many intermediate pillow blocks to insure subcritical operation. To eliminate this stumbling block, future mechanical power transmission systems will operate in the supercritical or "hypercritical" speed range.

The hypercritical system is shown in Fig. 5 and operates above the second critical speed. The design of this type of system requires special attention to the kinematic properties of the shaft as compared to first and possibly second critical speed systems now in existence which are specially balanced or amplitude limited and usually developed on a trial and error basis. The hypercritical system behaves much like a violin string vibrating at higher harmonics. Standing waves are developed and systems behavior is quite predictable. To achieve stable operation it is only necessary to absorb dynamically the vibrating energy content of the shaft at each critical speed through which the system must operate to reach the operating speed and within the operating speed range if necessary. Vibration energy absorption and system tuning is accomplished with damped bearing supports carefully designed for mass, spring rate, viscous damping, and location.

The tremendous advantage of this type of system is that only one or at the most two lightweight damped bearing supports, specifically located along the shaft, will satisfactorily carry long systems operating to the fifteenth or higher critical speeds. Flexible couplings are suggested at each end to accommodate misalignment of the associated equipment. The basic equations for bending and shear stresses discussed for subcritical systems still apply; however, the method for determining the physical properties of the damper and its location is beyond the scope of this paper.

Actual hypercritical systems have been operated through their fifteenth critical speed with no difficulty, as illustrated in Fig. 6. Special prototype dampers were developed and are being used for this program. The over-all system operation is surprisingly vibration free and operates more smoothly than conventional systems.

Weight Tradeoff Studies: Subcritical Shaft Systems

This section illustrates one suggested method of performing preliminary weight tradeoff studies for two of the forementioned systems. For the purpose of comparing the weights of all three systems three basic assumptions are made: 1) the tubing itself is steel, thin walled, with the o.d. stepped up to fit the i.d. of a bearing and engine in face spline-(curvic) type connections; 2) all intermediate bearings are single row, light series (deep groove) "Conrad" type, grease lubricated with a spherical o.d. when applicable; and 3) axial static and dynamic misalignment are either negligible or can be accommodated by the flexible couplings.

System A: Flexible Couplings at Ends Only with Bending in Shafting (Fig. 1)

In determining the shaft system weight vs operating speed for this type of system, the following procedure is suggested:

1) Determine the continuous operating, intermittent, and static torque and deflection requirements of the system at the various operating speeds being considered.

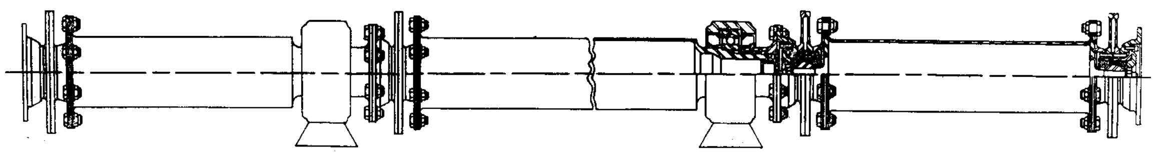

Fig. 3 Subcritical, type B, flexible couplings at each support.

2) Choose rough tubing o.d.'s for various bearing sizes and calculate the limiting speed for each bearing size. The limiting speed being dependent upon the max "*DN*" value desired. (D = bearing bore, mm; N = speed, rpm).

3) Determine "infinite life" fatigue stress levels in bending and torsion for the shafting materials being considered (include appropriate safety factors).

4) Determine the torsional shear stresses at various actual tube wall thicknesses for the rough tubing sized in step 2 using formula (1). (Formulas used are listed at the end of this section.)

5) Tabulate, at the system speeds being considered, the various tube o.d.'s and wall thicknesses that will meet the *DN* limits of step 2 and the torsional shear limits of step 3. Eliminate sizes for which curvic face splines would be marginal. (Fabrication and handling problems will probably limit minimum wall thickness of steel tubing to about 0.040 in.)

6) Check the tubing of step 5 for the critical buckling stress per formula (2) and eliminate sizes as necessary.

7) For all tubing sizes, thicknesses, and speeds noted in step 6, eliminate all but the thinnest wall thickness of each tube o.d. at each speed studied.

8) For the shaft system span in consideration and using the tubing sizes of step 7, find the bending stress in the tubing per formula (3) for various numbers of pillow blocks. (When figuring the deflection in each span, be sure to include all misalignments which cause the shafting to bend, i.e., radius of curvature of structure, installation tolerances, dynamic movements of driving, and driven members, etc.)

9) Compare the calculated bending stresses of step 8 with stress limit of step 3 for each number of pillow blocks chosen, and eliminate system configurations as necessary.

10) Calculate the approximate first critical speed of each configuration of step 8 per formula (4). (In multispan shaft systems where each span is of equal length and ends are considered simply supported at flexible coupling and spherical bearing, the critical speed of the system is approximately equal to the critical speed of any one simply supported section.)

11) Assuming that operating speed should not exceed $\frac{2}{3}$ of the first critical speed, tabulate the various shaft system configurations (i.e., operating speed vs shaft size and number of pillow blocks) pinpointed from steps 8–10.

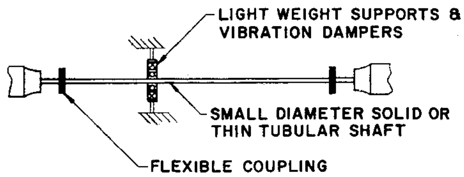

Fig. 5 Hypercritical system schematic.

12) Calculate the weight of each system of step 11, including the estimated weight of all bearing supports, flexible couplings, and intertubing connections.

13) Check the bearing loadings for each system, considering component imbalance, system weight, and bending reactions, to assure that the bearings meet the operating life requirements.

Formulas

Torsional Shear Stress in Tubing

$$S_s = 0.637\ Tr/(r^4 - r_i^4) \tag{1}$$

Critical Buckling Shear Stress (Ref. 1, Chap. 14, p. 317, Case 28)

$$S_s' = 0.272\mathrm{E}\ (t/r)^{3/2}/(1 - \mu^2)^{3/4} \quad \text{for } l/r \geqq 7.9 \tag{2}$$

Bending Stress in Shafting (Maximum) (Ref. 1, Chap. 8, p. 100, Case 1)

$$S_b = 3y\mathrm{E}r/l^2 \tag{3}$$

Critical Speed

$$N_c = (18.88 \times 10^6/l^2)(r^2 + r_i^2) \tag{4}$$

System B: Flexible Couplings at Each Support and at End Points (Fig. 3)

In determining the weight of this type of shaft system vs operating speed, the procedure used is very similar to that of system A. Since there is a flexible coupling at each intermediate support, there will be no requirements for shaft bending. Furthermore, it will not be necessary to have the shafting o.d. slip through the bearing i.d., as it must be adapted to the flexible coupling at each bearing. It will not be necessary to check the *DN* values of many different size bearings,

Fig. 4 Hiller OH-5A tail rotor system.

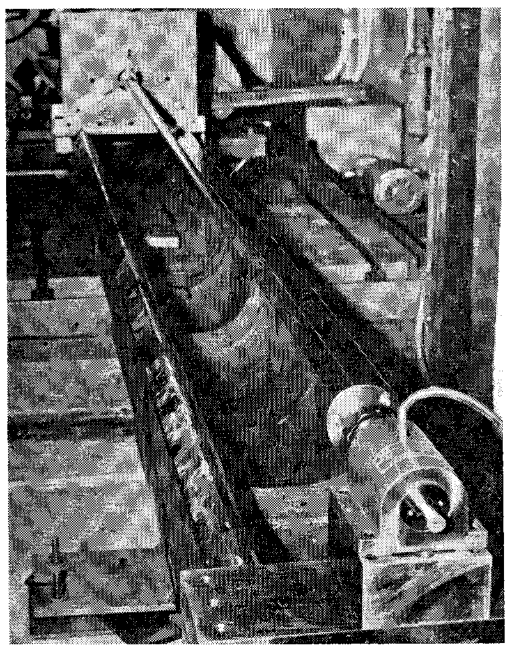

Fig. 6 Hypercritical test stand.

Table 1 Weight Tradeoff Studies

Speed rpm	Tubing size o.d. × wall, in.	Pillow blocks no.	Pillow blocks lb (ea.)	Flex couplings no.	Flex couplings lb (ea.)	Total system weight, lb[a] A	B	C
4,000	1.7 × 0.040	2	2.6	2	3.0	21.0	...	...
4,000	1.5 × 0.060	2	2.25	2	3.0	22.3	...	...
4,000	3.25 × 0.040	1	2.25	3	3.0	...	28.2	...
4,000	1.63 × 0.040	2	2.25	4	3.0	...	27.7	...
4,000	1.56 × 0.055	1	2.0	2	3.0	...	...	19
						...	...	...
7,000	1.5 × 0.040	3	2.25	2	2.65	20.7	...	...
7,000	1.3 × 0.040	4	2.0	2	2.65	21.2	...	...
7,000	2.75 × 0.040	2	2.0	4	2.65	...	29.4	...
7,000	1.3 × 0.046	1	1.75	2	2.65	...	...	15.1
						...	...	...
10,000	1.1 × 0.040	4	1.7	2	2.25	17.9	...	...
10,000	4.0 × 0.040	2	1.7	4	2.25	...	31.3	...
10,000	2.25 × 0.040	3	1.7	5	2.25	...	29.9	...
10,000	1.375 × 0.040	4	1.7	6	2.25	...	31.8	...
10,000	1.15 × 0.040	1	1.7	2	2.25	...	...	12.7

[a] Includes weight of tubing and connecting hardware.

since any practical size that will carry the loads can be utilized. The following procedure is suggested:

1) Using the same operating torques, misalignments, and speeds, determine various shaft sizes (o.d. and wall thickness) that will meet the torsional shear stress requirements of system A, step 3. The minimum practical wall thickness will again be about 0.040 in. and the maximum practical shaft o.d. will be about $4\frac{1}{2}$ in.

2) Check the interconnecting tubing for critical buckling, as in system A.

3) Determine required number of pillow blocks for each system to assure operation below first critical, as in system A.

4) Estimate weight of systems including all flexible couplings, bearing supports, and interconnecting connectors.

5) Check bearing loading.

Sample Weight Studies

A sample weight tradeoff study of system types A, B, and C was done for the following theoretical power transmission system requirements. Horsepower requirements are 500 continuous, 750 intermittent, and 1000 static. Speed requirements are to be determined (checked at 4000, 7000, and 10,000 rpm). Length requirements are 120 in. For misalignment, the following are continuous duty requirements and are considered to be additive: 1) airframe = 1500 in. uniform radius of curvature; 2) pillow blocks = ±0.062 in. parallel; and 3) end coupling = ±1.5° angular. Axial misalignment is considered negligible.

In addition to the preceding operating requirements, and as a starting point, a maximum *DN* of 300,000 was used for all bearings, and a maximum shear and bending stress of 50,000 psi each was used for all interconnecting tubing. The final *DN* and stress values are considerably lower. It is suggested that the values used to calculate an actual system be based upon past experience and special development tests as necessary. Table 1 illustrates the various system configurations developed for this sample power transmission requirement, and compares the weight of all three systems at the three speeds.

When evaluating the various shaft systems in Table 1 no one best system or systems speed can be suggested until the

Table 2 Performance of Single Diaphragm

o.d., in.	Single diaphragms, continuous duty: Rated angle, deg.	Rated torque, in-lb	Axial deflection, ±0.001 in.	Approx. width, in.	Approx. weight, lb.	Limit torque, in.-lb	Bending spring rate, in.-lb/deg	Torsional spring rate, in.-lb/deg	Speed cont. duty, rpm
2.75	0.1	7300	1	0.27	0.15	21,900	3836	93,100	67,800
	0.25	5250	2	0.22	0.11	15,750	246	37,200	67,800
	0.5	1620	4	0.16	0.04	4860	30.7	18,600	67,800
	0.75	500	6	0.14	0.03	1500	9.1	12,400	67,800
3.5	0.1	12,750	1	0.31	0.30	38,250	7300	171,000	54,700
	0.25	10,000	3	0.28	0.20	30,000	468	70,800	54,700
	0.5	3150	5	0.17	0.07	9450	58.5	35,400	54,700
	0.75	920	8	0.15	0.05	2760	17.3	23,800	54,700
4.6	0.1	32,400	1	0.47	1.05	97,200	17,500	425,000	40,900
	0.25	24,000	3	0.36	0.46	72,000	1123	169,800	40,900
	0.5	7400	7	0.18	0.18	22,200	140	84,900	40,900
	0.75	2200	11	0.16	0.17	6600	41.6	56,600	40,900
5.6	0.1	46,800	2	0.50	1.40	140,400	27,040	675,000	35,350
	0.25	37,100	4	0.45	0.80	111,300	1735	262,000	35,350
	0.35	24,700	6	0.29	0.46	74,100	514	175,000	35,350
	0.50	11,500	8	0.27	0.36	34,500	216	131,000	35,350
	0.75	3400	12	0.27	0.34	10,200	64	87,500	35,350
6.75	0.1	84,000	2	0.55	2.25	252,000	50,400	1,221,000	29,000
	0.25	69,000	5	0.45	1.40	207,000	3230	489,000	29,000
	0.5	21,300	10	0.37	0.7	63,900	403	244,000	29,000
	0.75	6500	15	0.35	0.6	19,500	119	158,000	29,000

effect of changing speed and adding pillow blocks is added to the entire airframe and power transmission system; however, the following is noted.

1) In the sample problem, shaft systems of type A (bending in shafting) decreased in system weight as speed increased. Increasing system speed at constant horse power decreased torque and, consequently, lightened torque-carrying members sufficiently to more than offset the weight increase due to additional intermediate supports that are necessary at the increased operating speeds. This is generally true with this type of system.

2) The shaft systems of type A are approximately $\frac{1}{3}$ lighter than comparable systems of type B. This is true, in most instances, for low to moderate power transmitting requirements. In high power transmitting systems with high misalignment requirements, it may be impossible to use a power transmission system with bending in the shafting, because of excessive bearing loading due to bending reactions.

3) In adding together the total number of components of systems A and B, at any given speed, the systems with shaft bending (type A) have fewer total components (i.e., pillow blocks, flexible couplings, adapter joints) and therefore better reliability and less maintenance requirements.

4) Type B systems show a slight increase in system weight with increasing operating speed.

5) Type C "hypercritical" shaft systems show considerable weight savings in the high speed ranges.

Conclusions

In general, the lightest weight system is obtained by operating at the highest practical speed and making the most efficient use of materials by eliminating intermediate couplings and using the elastic properties of the interconnecting shafting. High efficiency can be obtained by eliminating sliding and rolling surfaces that require lubrication and create heat that must be dissipated, and which represents a direct reduction in over-all efficiency. The elimination of these types of surfaces also increases the maintenance and overhaul periods.

The technology of designing a subcritical mechanical transmission system is well established; however, the final choice of a subcritical system comes only after its support requirement, and its effect on associated equipment, are integrated with the entire vehicle.

As the power requirement for future vehicles increases, mechanical systems are an absolute necessity because of their high over-all efficiency. The trend for future systems will be to the "hypercritical" because of their light weight and simplicity.

Appendix

This section discusses some of the major components noted within the text, and includes some basic design parameters, such as tables and formulas, for approximating their size and weight.

Flexible Couplings

The flexible couplings used in V/STOL and GEM type vehicles must be lightweight and reliable. Maintenance should be easily performed and the maintenance periods must be as long as practical. Lubrication, backlash, constant velocity ratio, axial, angular, torsional, and lateral spring rates are also considerations in the over-all design.

The contoured, diaphragm-type, flexible coupling fulfills most of these requirements, and it definitely follows the theme of efficient material usage. This type of coupling consists of one or more thin discs that are contoured to a specific shape between the rim and the hub that is almost hyperbolic, as contrasted to discs that have flat or tapered sides, or washers that are flat sided. The hyperbolic contour results in approximately uniform bending and torsion stresses and extremely efficient utilization of material. All angular movements occur as flexing in the contour or hyperbolic section, thereby eliminating lubrication requirements while providing constant velocity ratio with maximum radial stiffness. The discs or diaphragms are assembled in series, each disc proportionally sharing the total misalignment. The number of diaphragms can be varied to attain the desired operating angular misalignment without changing the operating stresses of any one diaphragm element.

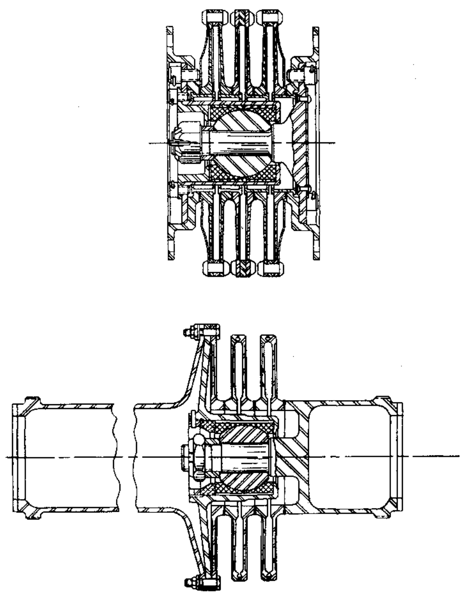

Fig. 7 Riveted vs welded construction.

The chart illustrated in Table 2 gives the performance parameters for various sizes of contoured diaphragms. It should be noted that the values specified in this table are approximate and are for single diaphragms.

Figure 7 illustrates the difference between riveted construction and welded construction. Figure 8 illustrates the use of typical end flanges and also ball and socket joints. The ball and socket joints are sometimes required to increase the lateral spring rate of the assembly. The ball joint also

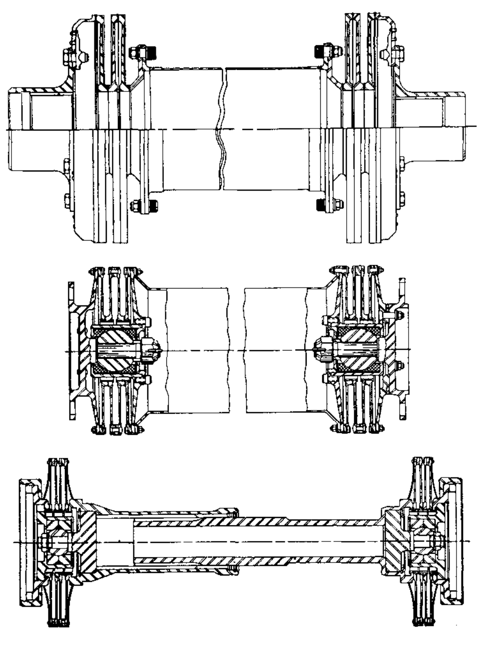

Fig. 8 Typical end flanges and ball socket joints.

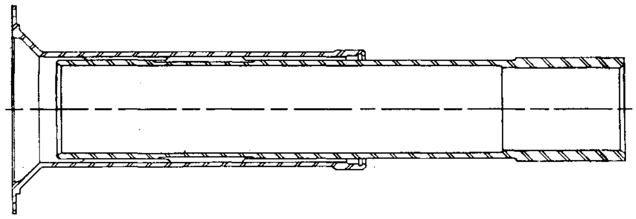

Fig. 9 Sliding telescoping splines.

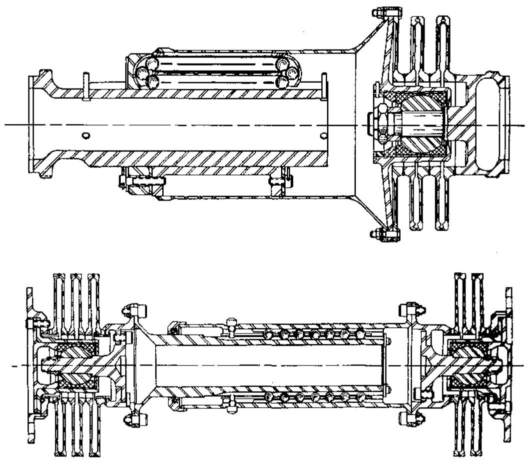

Fig. 10 Rolling telescoping splines.

restricts axial movements of the diaphragms when telescoping splines are used in conjunction with the diaphragm coupling assemblies.

Splines

If large axial movements from dynamic operation or setup tolerances are present in a mechanical power transmission system, it is sometimes impossible to eliminate splines. Splines add weight and reduce the over-all system reliability and maintenance periods. Lubrication is also required to keep wear and fretting corrosion under control.

There are two basic types of splines, i.e., sliding and rolling, and each has its advantages and disadvantages. Sliding splines are illustrated in Fig. 9 and can be manufactured in either the involute or the square tooth form. Rolling splines or ball splines are illustrated in Fig. 10. Two types of ball splines are presently in use. The recirculating type is used when it is necessary to accommodate large axial movements under load. If large movements are required for only installation

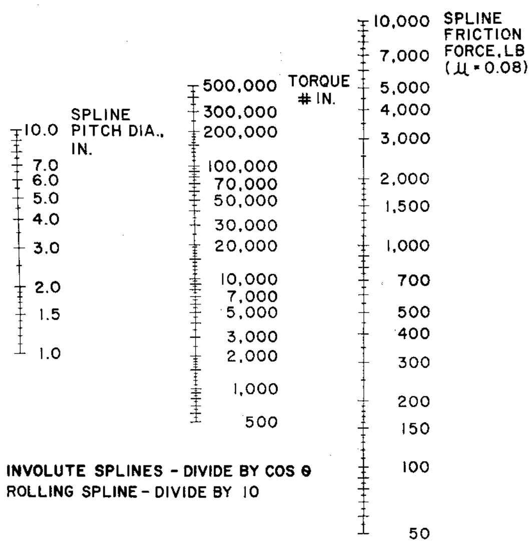

Fig. 11 Spline forces.

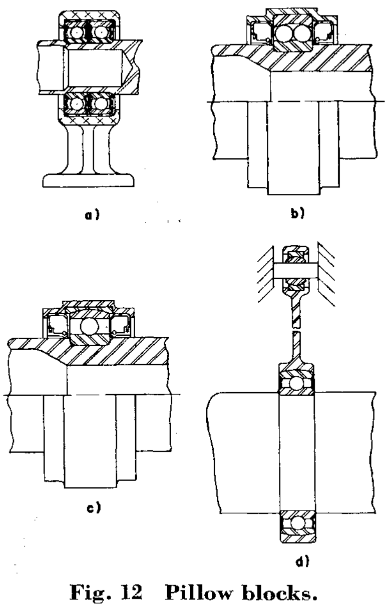

Fig. 12 Pillow blocks.

and removal at no load, then the noncirculating type is used, letting the balls skid in the races during installation. Special fits and preload grooves are also used to reduce fretting and wear.

Sliding Splines

Figure 11 illustrates the forces that sliding splines can cause in the system. The friction force is based on a coefficient of 0.08 with grease lubrication and square teeth. To estimate the weight it is necessary to approximate the actual size of the spline.

Rolling Splines

Rolling or ball splines have about $\frac{1}{10}$ the coefficient of friction of sliding splines, therefore, for estimating purposes, Fig. 11 may be used by dividing the force by 10. To estimate the weight of a rolling spline, it must be partially designed and a decision made regarding circulating and noncirculating. For grease lubrication, it is recommended that the mean Hertz stresses (Ref. 1, Chap. 13, p. 289, case 8) between the ball and the arch type groove be kept below 265,000 psi at rated torque and 475,000 psi at maximum static torque. The number of active ball races can be varied from three to six and, if extreme spacing accuracy and proper design is attained, it can be assumed that all balls and races are being loaded uniformly.

In all cases, extreme care should be taken to locate the splines in an area where the system bending movements are low. If bending moments (overturning couples) are present, their effect on the mean Hertz must be considered.

When sliding or rolling splines are incorporated in a shaft system, a detailed critical speed study must be done on the system to assure operation below the first critical speed.

Pillow Blocks

Figure 12 illustrates a few basic types of pillow blocks. Figure 12a is the fixed type and is used when no angular misalignment is required by the bearing, i.e., flex coupling on each side. Figures 12b and 12c have internal self-aligning capabilities and Fig. 12d has both self-aligning and axial movement capabilities.

Reference

[1] Roark, R. J., *Formulas for Stress and Strain* (McGraw-Hill Book Company, Inc., New York, 1954), 3rd ed.